Temporary Trade Barriers: When Will They End?

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Abstract

Under WTO law, temporary trade barriers such as antidumping duties must be removed after a period of four or five years. However, under rather vague legal conditions, they can be renewed. This institutional environment provides the structure to carefully study the determinants of the *length* of protection. We present a theoretical political-economy model of the renewal process and find that the probability a temporary trade barrier will be renewed decreases in the level of trade agreement tariffs and is also likely to decrease in the level of the anti-dumping duties. An increase in an industry's profitability is shown to increase the probability of renewal also increases. Lobbying effort also increases the probability for a fixed strength of the lobby, while the strength of the lobby itself has more nuanced effects on the renewal probability.

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1 Introduction

Today's world trading system is largely governed by a system in which countries commit to tariff bindings and yet raise their tariffs above those bindings through a variety of temporary trade barriers on a not-infrequent basis (Bown 2011). A significant literature has explored the question of which industries receive protection through temporary trade barriers and under what conditions this happens. This project asks a related but distinct question: given that a product receives protection, what determines the duration of the protection it receives?

This question can be addressed in the context of renewals of trade remedies under the WTO agreements. Normally, temporary measures such as anti-dumping and counter-vailing duties are imposed with a sunset

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provision that is subject to renewal. Conditional on a temporary trade barrier being granted and applied, one would like to know what determines whether a renewal order is granted so that the measure continues in force. In fact, this very particular institutional environment allows us to examine questions about the determinants of protection that are usually much more difficult to answer by holding fixed some of the key elements.

In many countries, the crucial actors in the imposition of temporary trade barriers are advocates for the industry that would be protected and some constellation of administrative bodies that have been granted authority to make decisions about whether a given temporary trade barrier is WTO-legal. For ease of exposition and because I would ultimately like to test this theory using U.S. data, I will refer to the actors in this model as industries and the government, and I will focus on antidumping duties because antidumping duties are both imposed and renewed most frequently and thus provided the largest sample size.

In this model, there are two phases of interaction between any given industry seeking protection and the government. In both cases, I assume that all actors take most-favored-nation (MFN) tariffs as given.

In the first phase, when no temporary trade barrier is in place, an industry exerts effort to convince the government to enact a temporary trade barrier for its product. The government must then decide whether to maintain the MFN tariff or impose the temporary trade barrier. Importantly, we assume that the decisions of the government are not deterministic, with the amount of uncertainty varying by industry.

Given that the industry receives temporary protection in the first phase of interaction and the protection remains in place until expiry, there will be another phase of interaction when the trade barrier expires, here modeled as five years later to match the typical five years sunset provision for antidumping duties under the WTO. At this point, the interaction is similar to that during the first phase, with the industry first exerting effort and then the government deciding whether to extend the temporary trade barrier or to revert to the MFN tariff. The essential strategic difference is that at this point the level of the temporary trade barrier is taken as exogenously set at the level determined in the first round of interaction. It is the second phase that is modeled in the current paper, with the addition of the first phase acknowledged as a very important and imminent extension.

An industry's decision about how much, if any, effort to exert depends on a number of factors. Among them are the gap between the applied tariff it faces and the protection it would receive under a temporary trade barrier, the cost of seeking the temporary trade barrier, and, crucially, the probability that its request will be granted. That is, the industry's incentives to seek protection, and the intensity with which it does so, depend on how much uncertainty it faces in the government's decision-making process.

Thus we turn to the question of how the government decides whether to grant a temporary trade barrier. The imposition of a temporary trade barrier, or continuation of one already in place, is not in general costless. We might assume then that the government only grants renewals when it finds the benefits outweigh the costs.

If the government's rulings are uncertain from the point of view of industry, it must be that the industry cannot perfectly predict the outcome of any given proceeding. That is, the industry does not know with precision how the government weighs the costs and benefits of granting protection.

There are several possible sources of this uncertainty. The industrial group may not be able to predict directly the quality of evidence the government requires in order to be willing to provide protection through a temporary trade barrier. Indirectly, this may reflect the government's preferences about the costs of disputes and retaliation by trade partners. Industry may also not be fully informed about how the government weighs the potential harms to the industry versus those to up- and down-stream industries, consumers and trading relations. This may be derived from factors such as the political influence of the industry, how central it is to the economy, and whether there is active lobbying by producers downstream of the product in question.

Interestingly, there are very few stipulations in the General Agreement on Tariffs and Trade (GATT) / World Trade Organization (WTO) to guide the renewal process. WTO-level litigation has not provided significant additional structure, so governments appear to have significant latitude in their decision-making process regarding renewals of temporary trade barriers. These decisions are often made via complex processes, adding to uncertainty.

In the United States, for the renewal of antidumping duties, the Department of Commerce must initiate a renewal proceeding and determine whether dumping would continue of the duties are not renewed. The International Trade Commission determines whether injury would recur or continue. These bodies are likely susceptible to the influence of lobbying both directly from industry and consumer groups and indirectly via other political bodies such as Congress as well as other parts of the Executive branch.

Given this complicated decision-making process with few guidelines provided at the international level, I present a theoretical model to guide our thinking about what elements impact renewal decisions. I find that the probability that antidumping duties will be renewed is invariant to foreign tariffs, decreases in most-favored nation tariffs and is likely to decrease in the level of the anti-dumping duties. The probability increases in lobbying effort, but the overall impact of the strength of the lobby depends on whether the positive direct effect outweighs the negative indirect effect on the lobby's incentives. If the industry becomes more (less) profitable in the sense that the gap between profits under the antidumping duties and profits under the trade agreement tariffs increases (decreases), the probability of renewal also increases.

There is much to be learned about the workings of the world trading system by examining the duration of deviations from base tariff commitments. After extending this model to include the stage at which antidumping duties are set, I plan to take this model to a data set of renewal decisions for anti-dumping measures.

2 The Model

2.1 The Basic Setup

I begin by describing the basic economic setting within which trade occurs. It is a three-good model with two countries: home (no asterisk) and foreign (asterisk). In each country, preferences are linear in good N, which is denoted the numeraire, while the demand functions for X and Y are assumed strictly decreasing and twice continuously differentiable. The demand functions for X and Y are taken to be identical and written $D(P_i)$ in home and $D(P_i^*)$ in foreign. P_i (P_i^*) denotes the home (foreign) price of good $i \in \{N, X, Y\}$.

Good N is produced with labor alone so that $Q_N = l_N$. I assume the aggregate labor supply is large enough to ensure that the output of good N is enough to guarantee balanced trade. The supply functions for good X are $Q_X(P_X)$ and $Q_X^*(P_X^*)$ and are assumed strictly increasing and twice continuously differentiable for all prices that elicit positive supply. For any such P_X, I assume $Q_X^*(P_X) > Q_X(P_X)$ so that the home country is a net importer of good X. The production structure for good Y is symmetric, with demand and supply such that the economy is separable in goods X and Y. The production of goods X and Y requires labor and a sector-specific factor that is available in inelastic supply and is non-tradable so that the income of owners of the specific factors is tied to the price of the good in whose production their factor is used.

For simplicity, I assume each government's only trade policy instrument is a specific tariff on its import-competing good: the home country levies a tariff τ on good X while the foreign country applies a tariff τ^* to good Y. Local prices are then $P_X = P_X^W + \tau$, $P_X^* = P_X^W$, $P_Y = P_Y^W$ and $P_Y^* = P_Y^W + \tau^*$ where a W superscript indicates world prices. Equilibrium prices are determined by the market clearing conditions

$$M_X(P_X) = D(P_X) - Q_X(P_X) = Q_X^*(P_X^*) - D(P_X^*) = E_X^*(P_X^*)$$

$$E_Y(P_Y) = Q_Y(P_Y) - D(P_Y) = D(P_Y^*) - Q_Y^*(P_Y) = M_Y^*(P_Y^*)$$

where e.g. M_X are home-county imports and E_X^* are foreign exports of good X. The price of the numeraire is equal to one in both countries and on the world market.

P_X^W and P_Y^W are decreasing in τ and τ^* respectively, while P_X and P_Y^* are increasing in the respective domestic tariff. This gives rise to a terms-of-trade externality. Profits in a sector are increasing in the price of its good and also in the domestic tariff. This fact, combined with the assumptions on specific factor ownership, motivates political activity by the import-competing lobby.

In the home country, there is a government agency that has the power to renew (or not) antidumping duties. To focus attention on protectionist political forces, highlight the model's central mechanisms and minimize the number of actors that must be modeled, we assume that only the import-competing industry is politically-organized and that it is represented by a single lobbying organization.[1] Policy-makers and lobbyists in the foreign country are assumed to be passive.

Taking the trade-agreement tariffs $\boldsymbol{\tau^a} = (\tau^a, \tau^{*a})$[2] and the antidumping duty level τ^{ad} as given, the import-competing firms jointly attempt to persuade the government agency to renew the antidumping duties by choice of lobbying effort e.

After this, uncertainty about the government agency's preferences is resolved.

All players simultaneously observe the realization of the random variable θ that represents this uncertainty. The political stage concludes with the agency making a choice to renew the antidumping duties or let them expire. In the event that the antidumping duties expire, the home country's trade policy reverts to the trade agreement tariff τ^a. Finally, producers and consumers make their decisions.

Although political uncertainty is important in informing the lobby's behavior, the player that has more information moves last in the game so subgame perfect Nash equilibrium is the appropriate solution concept.

2.2 Preferences

With foreign not active in setting policy and the structure the economy symmetric and fully separable, I focus on the home country and the X-sector, which is the only politically active sector. The government agency's welfare is given by

$$W_G = CS_X(\tau) + \gamma(e,\theta) \cdot \pi_X(\tau) + CS_Y(\tau^*) + \pi_Y(\tau^*) + TR(\tau) \tag{1}$$

where CS is consumer surplus, π are profits, $\gamma(e,\theta)$ is the weight placed on profits in the import-competing industry, and TR is tariff revenue.[3] I model the decisions of the government agency as being taken by a median actor with the weight the median actor places on import-competing industry profits affected by the level of lobbying effort e and a random variable θ. I make the following assumptions on $\gamma(e,\theta)$:

Assumption 1. *$\gamma(e,\theta)$ is increasing and concave in e for every $\theta \in \Theta$.*

Assumption 2. *$\gamma(e,\theta)$ is increasing in θ.*

Assumption 3. *The pdf of the induced distribution on $\gamma(e,\theta)$ is weakly increasing in e.*

Assumption 1 means that the government agency favors the import-competing industry more the higher is its lobbying effort, but that there are diminishing returns to lobbying activity. It rules out higher effort making

[1] The model extends easily to the case of multiple lobbies.

[2] I use the convention throughout of representing a vector of tariffs for both countries (τ, τ^*) as a single bold $\boldsymbol{\tau}$.

[3] Labor income l could also be included in government welfare. I omit it because its inclusion alters none of the results and only serves to complicate the exposition.

lower weights more likely and that the structure of uncertainty changes with increasing effort so that higher weights become more likely at an accelerating pace. Assumption 2 simply provides for an intuitive labeling so that larger realizations of θ increase the value of the political economy weight. Assumption 3 is a restriction either on the distribution of uncertainty or on how it enters $\gamma(e, \cdot)$ so that higher values of the political economy weight become more likely at higher values of lobbying effort.

Given its expectations and the government agency's preferences, the home lobby chooses its lobbying effort e to maximize the welfare function:

$$\mathbb{E}\left[U_L\right] = \Pr\left[\text{AD Renewal}\right] \pi_X(\tau^{ad}) + \{1 - \Pr\left[\text{AD Renewal}\right]\} \ \pi_X(\tau^{a}) - e \tag{2}$$

where $\pi(\cdot)$ is the current-period profit.

3 Main Results

The government agency will renew the antidumping duty if the median actor's utility from the antidumping duty is higher than his utility from the trade agreement tariffs, i.e. if

$$W_G(\tau^{ad}, \tau^{*a}, \gamma(e, \theta)) > W_G\left(\boldsymbol{\tau^a}, \gamma(e, \theta)\right) \tag{3}$$

The outcome of the decision on whether or not to renew the antidumping duties is not known to *any* player until the uncertainty over the identity of the median governmental actor is resolved at the moment the decision is made. I represent the probability that antidumping duties are renewed as:

$$\begin{aligned} r(e, \boldsymbol{\tau^a}, \tau^{ad}) &= \mathbb{E}_{\gamma|e} \mathbf{1}[W_G(\tau^{ad}, \tau^{*a}, \gamma(e, \theta)) > W_G\left(\boldsymbol{\tau^a}, \gamma(e, \theta)\right)] \\ &= \Pr[W_G(\tau^{ad}, \tau^{*a}, \gamma(e, \theta)) > W_G\left(\boldsymbol{\tau^a}, \gamma(e, \theta)\right) |e] \end{aligned} \tag{4}$$

The lobby chooses its level of effort as a function of the trade agreement tariffs and the antidumping duties, given the implications of that choice on the government agency's probability of renewing the duties. In effect, the lobby maximizes probability-weighted profits net of effort:

$$\max_e r(e, \boldsymbol{\tau^a}, \tau^{ad}) \pi(\tau^{ad}) + [1 - r(e, \boldsymbol{\tau^a}, \tau^{ad})] \pi(\tau^a) - e$$

The first order condition shows that the lobby balances the cost of an extra dollar of expenditure with the higher profits from anti-dumping duties weighted by the increase in the probability of receiving the extension of the antidumping duties:

$$\frac{\partial r(e, \boldsymbol{\tau^a}, \tau^{ad})}{\partial e}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right] = 1 \tag{5}$$

Result 3 below and the fact that antidumping duties are necessarily larger than trade agreement tariffs ensure the second order condition. To guarantee an interior solution, we need

$$\frac{\partial r(0, \boldsymbol{\tau^a}, \tau^{ad})}{\partial e}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right] > 1. \tag{6}$$

It may be that either the antidumping duty is too low or the marginal impact of the first lobbying dollar is too small for it to be in the lobby's interest to exert effort. The main results of the paper only hold when the marginal impact of the first lobbying dollar on the renewal probability is sufficiently high to make engaging in the political process worthwhile for the lobby.

I represent the total probability that the antidumping duties will be renewed as $R(\boldsymbol{\tau^a}, \tau^{ad}) = r(e(\boldsymbol{\tau^a}, \tau^{ad}), \boldsymbol{\tau^a}, \tau^{ad})$ where $e(\boldsymbol{\tau^a}, \tau^{ad})$ is the best response function implicit in Equation 5, the lobby's first order condition. We next examine the question of how the total renewal probability varies as both a direct and indirect function of both trade agreement tariffs and antidumping duties, as well as industry profitability and political influence. Each result must take into account that the variable of interest can have both a direct effect on the renewal probability and an indirect effect through lobbying incentives.

3.1 Home Trade Agreement Tariffs

Let's look first at the direct effect of changing τ^a on the renewal probability. For any effort level e, when the trade agreement tariff is very low, only a small set of realizations of θ associated with the lowest values of $\gamma(e, \theta)$ will lead to the antidumping duties being re-approved, implying a high renewal probability. As the trade agreement tariff rises, larger values of $\gamma(e, \theta)$ are consistent with approving the trade agreement, so the set of θ's that lead to trade agreement approval is larger. That is, the renewal probability decreases as τ^a increases.

Lemma 1. *The direct effect of the home trade agreement tariff on the probability the government renews the antidumping duty is negative. Proof*

All proofs are in the Appendix.

The indirect effect is made up of two parts: the impact of raising the home trade agreement tariff on lobbying effort, and the impact of lobbying effort on the renewal probability. Starting with the question of how lobbying effort varies in the home trade agreement tariff, notice that raising the trade agreement tariffs decreases the benefit of a renewal in the trade agreement by raising trade agreement profits. This implies that lobbying effort is lower when trade agreement tariffs are higher.

Lemma 2. *Lobbying effort is weakly decreasing in trade agreement tariffs. Proof*

Turning to the second component of the indirect effect, it's useful to note that lobbying affects only the weight the government agency places on the profits of the import-competing industry. These profits are higher under antidumping duties than under the trade agreement tariff. Assumption 1 implies that the government becomes more favorably inclined—albeit at a decreasing rate—toward the higher antidumping duties and associated profits as lobbying increases and thus more likely to renew the antidumping duties.

Lemma 3. *The probability that the government agency renews the antidumping duties is increasing and concave in lobbying effort* $\left(\textit{i.e.}\ \frac{\partial r}{\partial e} \geq 0,\ \frac{\partial^2 r}{\partial e^2} \leq 0\right)$*. Proof*

Result 1 combines the direct and indirect effects to get the total effect of trade agreement tariffs on the renewal probability.

Result 1. *The total probability that the antidumping duties will be renewed is decreasing in* τ^a *(i.e.* $\frac{\partial R(\boldsymbol{\tau^a}, \tau^{ad})}{\partial \tau^a} \leq 0$*). Proof*

When τ^a is higher, the government agency becomes less likely to renew the antidumping duties, for three reasons. First, the government prefers a higher domestic tariff (Lemma 1); second, the higher tariff discourages lobbying (Lemma 2); and finally, the lower lobbying effort directly reduces the government's preferred tariff further (Lemma 3).

3.2 Foreign Trade Agreement Tariffs

Turning to the effect of the foreign trade agreement tariffs on the probability that the antidumping duties will be renewed, it is perhaps not surprising that there is no effect:

Result 2. *The total probability that the antidumping duties will be renewed is constant in the foreign trade agreement tariff. Proof*

Because the duties are a unilateral policy that we presume here are permissible under WTO rules, the decision to renew the duties or not does not change the level of the tariff applied by the foreign trading partner. That is, there is no incentive in terms of reciprocal treatment to deter the home government from renewing the duties.

3.3 Antidumping Duties

The level at which the antidumping duties are set is likely to be related to some of the variables under consideration here, as the duties are determined five (or more) years earlier through a process similar to the renewal process. Endogenizing the level of the antidumping duties, as well as the fact of their application in the first place, is planned as an important extension.

Even in the case where we take the antidumping duties as exogenous, the prediction is not clear cut. The direct effect is negative, as the government becomes less willing to protect the industries as the higher duties increasingly harm social welfare.

Lemma 4. *The direct effect of an increase in the antidumping duty on the probability the government renews the antidumping duties is negative. Proof*

However, the indirect effect of raising antidumping duties is ambiguous. Higher duties mean the return to a positive renewal decision is larger and thus the lobby has a larger incentive to exert effort; at the same time, the marginal impact of lobbying on the renewal probability falls as the antidumping duties increase and this blunts lobbying incentives. We can't say whether lobbying expenditure will rise or fall without knowing the magnitude of these two effects.

Lemma 5. *Higher antidumping duties may lead to either an increase or decrease in lobbying effort. Proof*

Because the direct effect of increasing antidumping duties to reduce the renewal probability, the only way the total effect of increasing antidumping duties leads to a larger probability of renewal is if the lobbying response is positive and large in magnitude.

Result 3. *The total probability that the antidumping duties will be renewed is most likely to decrease in the level of the antidumping duties but it can increase in the antidumping duties if the indirect effect is positive and large enough to outweigh the direct effect. Proof*

3.4 Political Weighting Function

We next examine the weight the median decision-maker places on the profits of the import-competing sector. The value of the political weight $\gamma(e, \theta)$ is endogenous to many of the decisions underpinning the equilibrium, but here we examine the effect of an exogenous shift in the weighting function itself, $\gamma(\cdot, \theta)$. The type of shift we analyze is one in which the value of $\gamma(\cdot, \theta)$ weakly increases for every value of e.

The direct effect is positive. This is quite intuitive: when the government places a higher weight on the industry's profits for a given lobbying effort, it is more likely to renew the antidumping duties because the industry's profits are now more important relative to social welfare.

Lemma 6. *The direct effect of an exogenous positive shift in the political weighting function $\gamma(\cdot, \theta)$ on the probability the government renews the antidumping duty is positive. Proof*

Perhaps not surprisingly, the lobby's incentives move in the opposite direction. A shift in $\gamma(\cdot, \theta)$ does not directly change the lobby's payoff. But it makes its marginal unit of effort more productive, so the lobby optimally reduces effort (i.e. the marginal cost of a unit of effort stays the same, so the lobby reduces effort when the increase in $\gamma(\cdot, \theta)$ improves the marginal benefit).

Lemma 7. *Lobbying effort is weakly decreasing in exogenous positive shifts of the political weighting function. Proof*

The total effect of the positive shift in $\gamma(\cdot, \theta)$ depends on which of the direct or indirect effects is larger.

Result 4. *The total probability that the antidumping duties will be renewed is increasing in $\gamma(\cdot)$ if the direct effect dominates and is decreasing in $\gamma(\cdot)$ if the indirect effect dominates. Proof*

3.5 Profitability of Import Competing Sector

Finally we turn to questions of the lobbying sector's profitability. Profitability can shift in many ways, and they can affect incentives in many ways. In terms of the impact on both the government and lobby's decisions around renewal of temporary trade barriers, it turns out that changes in the gap between the profits under the antidumping duties and under the trade agreement tariff are a sufficient statistic. So we examine any change to profitability that affects this gap in a clear way.

If profitability changes in such a way that the lobbying sector will now be even more profitable under the antidumping duties compared to the trade agreement tariff, the government becomes more likely to renew the antidumping duties. This shift improves the evaluation of the antidumping duties both in terms of the social welfare calculation and in terms of the lobby's benefit, but the lobby's benefit is weighted more heavily so the balance is tipped in favor of renewal.

Lemma 8. *The direct effect of a change in the import competing sector's profitability on the probability the government renews the antidumping duty has the same sign as the change in the gap between profits under antidumping duties and profits under the trade agreement tariff. Proof*

From the lobby's perspective, an increase in the profit gap increases its incentives to exert lobbying effort; at the same time, because the government's renewal probability hinges on how much it over-weights profits, it becomes more worthwhile to exert lobbying effort to influence the government's decision-making weight directly.

Lemma 9. *The effect of a change in the import competing sector's profitability on lobbying effort has the same sign as the change in the gap between profits under antidumping duties and profits under the trade agreement tariff. Proof*

The direct and indirect effect combine so that the change in total probability depends only on the change in the profit gap under the antidumping duties and the trade agreement tariff.

Result 5. *The total probability that the antidumping duties will be renewed increases (decreases) when there is an increase (decrease) in the gap between profits under antidumping duties and profits under the trade agreement tariff. Proof*

The bottom line: in order for the total probability of renewal to go up, it must be that the shift in profitability leads to an increase in the government's marginal benefit from protection as well as the returns from lobbying. To think about changes to profitability, we can imagine transformations of the (convex) profit function. Different increasing transformations could leave the profit function more or less convex than the original. Both the government and the lobby care about $\pi_X(\tau^{ad}) - \pi_X(\tau^{a})$—that is, cardinal information is important. So we must pay attention to how this difference changes when the profit function changes in order to make predictions about the impact on the renewal of antidumping duties.

4 Conclusion

This is very much work in progress. The next stage is to extend the model to include the initial decision to grant protection. This is important because of course the imposition of the original protective measure is related to the incentives to renew it, so the endogeneity should be taken into account.

What's more, we'd like to know why an industry that receives protection as a result of the original interaction concerning the imposition of the temporary trade barrier would fail to get it renewed—or, even more interesting, choose not to pursue a renewal when contacted by the Department of Commerce just before the antidumping duties expire. At the most basic level, if we assume the industry faces the same uncertainty and responds in the same way, it could get a different draw and thus a different outcome. Moreover, having the level of the temporary trade barrier set by the original interaction serves to reduce the industry's incentive to exert effort on the margin.

In addition, some factors that affect both the government's and the industry's decision-making can change significantly over the course of a five-year period. Most of these factors display at least some variation across industries.

Because a temporary trade barrier insulates an industry from competition, when receiving such protection, an industry should be able to increase profits. The amount that profits increase will vary by industry, and the decisions about how to use the extra profits will also plausibly vary. Some industries may use this opportunity to become more competitive and thus less interested in seeking protection while others may use the added profits to become more politically powerful and thus better able to gain future protection. These decisions, and the basic question of whether protection and technological upgrading are complements or substitutes, feeds back into the decision-making process of the government.

Another interesting dimension along which there is uncertainty for the government is in its evaluation of the probability with which trading partners will dispute or retaliate against a temporary trade barrier. This not only varies across temporary trade barriers, but may vary across time. In particular, the government can observe whether a dispute has been filed during the first five years of a temporary trade barrier and update its beliefs about future retaliation in a way that varies across industries.

All of this will then be taken to the data in the form of the World Bank's Temporary Trade Barriers Database.

5 Appendix

Proof of Lemma 1:

Substituting from Equation 1, Equation 4 can be re-written as

$$r(e, \boldsymbol{\tau^a}, \tau^{ad}) = \Pr[CS_X(\tau^{ad}) + \gamma(e,\theta)\cdot\pi_X(\tau^{ad}) + CS_Y(\tau^{*a}) + \pi_Y(\tau^{*a}) + TR(\tau^{ad}) > \\ CS_X(\tau^{a}) + \gamma(e,\theta)\cdot\pi_X(\tau^{a}) + CS_Y(\tau^{*a}) + \pi_Y(\tau^{*a}) + TR(\tau^{a})] \quad (7)$$

Rearranging, we have $r(e_b, \boldsymbol{\tau^a}, \tau^{ad}) =$

$$\Pr\Big[\tfrac{CS_X(\tau^{ad})+\pi_X(\tau^{ad})+CS_Y(\tau^{*a})+\pi_Y(\tau^{*a})+TR(\tau^{ad})-CS_X(\tau^{a})-\pi_X(\tau^{a})-CS_Y(\tau^{*a})-\pi_Y(\tau^{*a})-TR(\tau^{a})}{\pi_X(\tau^{a})-\pi_X(\tau^{ad})} + 1 \\ < \gamma(e,\theta)\Big] \quad (8)$$

The effect on the renewal probability is determined by the sign of the derivative of the left hand side of the inequality in Expression 8 with respect to τ^a; to show that the renewal probability is decreasing in τ^a, I must demonstrate that this derivative is positive. Labeling the left hand side of the inequality in Expression 8 $Z(\boldsymbol{\tau^a}, \tau^{ad})$ and the numerator of that expression $ZN(\boldsymbol{\tau^a}, \tau^{ad})$, the derivative of this quantity with respect to

τ^a is

$$\frac{\left(\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right)\left(\frac{\partial CS_X(\tau^a)}{\partial \tau^a} + \frac{\partial \pi_X(\tau^a)}{\partial \tau^a} + \frac{\partial TR(\tau^a)}{\partial \tau^a}\right) - ZN(\boldsymbol{\tau^a}, \tau^{ad})\frac{\partial \pi_X(\tau^a)}{\partial \tau^a}}{\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)^2}. \tag{9}$$

$\left(\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right)$ is always positive. Because the optimal unilateral tariff for large welfare-maximizing governments is positive (call it τ^O), $\left(\frac{\partial CS_X(\tau^a)}{\partial \tau^a} + \frac{\partial \pi_X(\tau^a)}{\partial \tau^a} + \frac{\partial TR(\tau^a)}{\partial \tau^a}\right)$ is increasing up to τ^O and decreasing above it. Thus the first summand is increasing up until τ^O and decreasing thereafter.

It is also positive over the remaining (τ^O, τ^{ad}). To see this, add $\left(\tilde{\Gamma} - 1\right)\frac{\partial \pi_X(\tau^a)}{\partial \tau^a}\left(\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right)$ to the first summand and subtract it from the second. For any particular value of $\tilde{\tau}^a$, one can choose the $\tilde{\Gamma}$ weight that would make $\tilde{\tau}^a$ the preferred unilateral tariff; this makes the derivative in the first summand zero. Having subtracted the same quantity from the second summand modifies the welfare difference in the second summand to be maximized at $\tilde{\tau}^a$ so that this term is always positive (inclusive of the leading negative sign).

Since the denominator is a squared term of a strictly positive quantity, the expression in Equation 9 is positive for all τ^a. ■

Proof of Lemma 2:

Proof is via the Implicit Function Theorem using the lobby's first order condition, Equation 5, referred to here as FOC_L.

$$\frac{\partial e}{\partial \tau^a} = -\frac{\frac{\partial FOC_L}{\partial \tau^a}}{\frac{\partial FOC_L}{\partial e}} = \frac{\frac{\partial r}{\partial e}\frac{\partial \pi(\tau^a)}{\partial \tau^a} - \frac{\partial^2 r}{\partial e \partial \tau^a}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right]}{\frac{\partial^2 r}{\partial e^2}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right]}$$

Beginning with the denominator: $\tau^{ad} > \tau^a$ and $\pi(\cdot)$ is increasing in τ so the expression is brackets is positive. $\frac{\partial^2 r}{\partial e^2}$ is negative by Result 3, so the denominator is negative.

Turning to the numerator: $\frac{\partial r}{\partial e}$ is positive by Lemma 3 below and $\frac{\partial \pi(\tau^a)}{\partial \tau^a}$ is positive by construction so the first term is positive.

We can rewrite $\frac{\partial r(e, \boldsymbol{\tau^a}, \tau^{ad})}{\partial \tau^a} = -\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a}, \tau^{ad}))}{\partial \tau^a} = -\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a}, \tau^{ad}))}{\partial Z(\boldsymbol{\tau^a}, \tau^{ad})}\frac{\partial Z(\boldsymbol{\tau^a}, \tau^{ad})}{\partial \tau^a} = -f_\gamma \frac{\partial Z(\boldsymbol{\tau^a}, \tau^{ad})}{\partial \tau^a}$, where $F_\gamma(Z(\boldsymbol{\tau^a}, \tau^{ad}))$ is the CDF of γ and $f_\gamma(Z(\boldsymbol{\tau^a}, \tau^{ad}))$ is the pdf of γ and $Z(\boldsymbol{\tau^a}, \tau^{ad})$ again represents the left hand side of the inequality in Expression 8.

Then $\frac{\partial^2 r}{\partial e \partial \tau^a} = -\frac{\partial}{\partial e}\left(\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a}, \tau^{ad}))}{\partial \tau^a}\right) = -f_\gamma \frac{\partial^2 Z(\boldsymbol{\tau^a}, \tau^{ad})}{\partial e \partial \tau^a} - \frac{\partial Z(\boldsymbol{\tau^a}, \tau^{ad})}{\partial \tau^a}\frac{\partial f_\gamma}{\partial e} = -\frac{\partial Z(\boldsymbol{\tau^a}, \tau^{ad})}{\partial \tau^a}\frac{\partial f_\gamma}{\partial e}$ where the above equality holds because $Z(\boldsymbol{\tau^a}, \tau^{ad})$ does not depend on e. The proof of Lemma 1 shows that $\frac{\partial Z(\boldsymbol{\tau^a}, \tau^{ad})}{\partial \tau^a} \geq 0$, so $\frac{\partial f_\gamma}{\partial e} \geq 0$ ensures that $\frac{\partial^2 r}{\partial e \partial \tau^a} \leq 0$. Thus $\frac{\partial e}{\partial \tau^a} \leq 0$. ■

Proof of Lemma 3:

The left side of the inequality in Expression 8, $Z(\boldsymbol{\tau^a}, \tau^{ad})$, does not depend on e. Thus we have $r(e, \boldsymbol{\tau^a}, \tau^{ad}) =$

$\Pr[Z < \gamma(e,\theta)] = 1 - F_\gamma(\gamma = Z)$ where $F_\gamma(\gamma = Z) = F_\theta(\theta = h^{-1}(\gamma, e))$ by the Change of Variables Theorem and Assumption 2 with $\gamma = h(e,\theta)$ giving the change of variable.

Then $\frac{\partial r}{\partial e} = -\frac{\partial F_\theta(\theta = h^{-1}(\gamma,e))}{\partial \theta}\frac{\partial h^{-1}(\gamma,e)}{\partial e} = -f_\theta(\theta)\frac{\partial h^{-1}(\gamma,e)}{\partial e}$ and $\frac{\partial^2 r}{\partial e^2} = -f_\theta(\theta)\frac{\partial^2 h^{-1}(\gamma,e)}{\partial e^2}$. Because $\gamma = h(e,\theta)$ is increasing and concave in e $\forall\theta$ by Assumption 1, its inverse is decreasing and convex $\left(\frac{\partial h^{-1}(\gamma,e)}{\partial e} \leq 0;\ \frac{\partial^2 h^{-1}(\gamma,e)}{\partial e^2} \geq 0\right)$ The pdf of θ is non-negative, so $\frac{\partial r}{\partial e} \geq 0$ and $\frac{\partial^2 r}{\partial e^2} \leq 0$. ■

Proof of Result 1:

$\frac{\partial R(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^a} = \frac{\partial r}{\partial e}\frac{\partial e}{\partial \tau^a} + \frac{\partial r}{\partial \tau^a}$. $\frac{\partial r}{\partial e}$ is non-negative by Lemma 3. $\frac{\partial e}{\partial \tau^a}$ is non-positive by Lemma 2. $\frac{\partial r}{\partial \tau^a}$ is non-positive by Lemma 1. Thus the entire expression is non-positive. ■

Proof of Result 2:

$\frac{\partial R(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{*a}} = \frac{\partial r}{\partial e}\frac{\partial e}{\partial \tau^{*a}} + \frac{\partial r}{\partial \tau^{*a}}$. $\frac{\partial r}{\partial \tau^{*a}}$ is zero because Expression 8, after simplification, is not a function of τ^{*a}. $\frac{\partial e}{\partial \tau^{*a}}$ is also zero because Equation 5 is not a function of τ^{*a}. Thus the entire expression is zero. ■

Proof of Lemma 4:

The effect on the renewal probability is determined by the sign of the derivative of the left hand side of the inequality in Expression 8 with respect to τ^{ad}; to show that the renewal probability is decreasing in τ^{ad}, I must demonstrate that this derivative is positive. Again labeling the numerator of that expression $ZN(\boldsymbol{\tau^a},\tau^{ad})$, the derivative of this quantity with respect to τ^{ad} is

$$\frac{\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)\left(\frac{\partial CS_X(\tau^{ad})}{\partial \tau^{ad}} + \frac{\partial \pi_X(\tau^{ad})}{\partial \tau^{ad}} + \frac{\partial TR(\tau^{ad})}{\partial \tau^{ad}}\right) + ZN(\boldsymbol{\tau^a},\tau^{ad})\frac{\partial \pi_X(\tau^{ad})}{\partial \tau^{ad}}}{\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)^2}. \tag{10}$$

$\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)$ is always negative. Because the optimal unilateral tariff for large welfare-maximizing governments is positive (call it τ^O), $\left(\frac{\partial CS_X(\tau^{ad})}{\partial \tau^{ad}} + \frac{\partial \pi_X(\tau^{ad})}{\partial \tau^{ad}} + \frac{\partial TR(\tau^{ad})}{\partial \tau^{ad}}\right)$ is increasing up to τ^O and decreasing above it. Thus the first summand is increasing up until τ^O and decreasing thereafter. Note it seems unreasonable for τ^{ad} to be smaller than τ^O but this can be proved for the general case so we will not assume it.

$ZN(\boldsymbol{\tau^a},\tau^{ad})$ is also always positive on $[0,\tau^O]$ since $\tau^{ad} > \tau^a$ and on this range τ^{ad} is closer to the optimum. Profits are increasing in τ^{ad}, so the second summand is positive on $[0,\tau^O]$. With a positive denominator, we thus have that the entire expression is positive on $[0,\tau^O]$.

It is also positive for all tariffs larger than τ^O. To see this, add $\left(\tilde{\Gamma} - 1\right)\frac{\partial \pi_X(\tau^{ad})}{\partial \tau^{ad}}\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)$

to the first summand and subtract it from the second. For any particular value of $\tilde{\tau}^{ad}$, one can choose the $\tilde{\Gamma}$ weight that would make $\tilde{\tau}^{ad}$ the preferred unilateral tariff; this makes the derivative in the first summand zero. Having subtracted the same quantity from the second summand modifies the welfare difference in the second summand to be maximized at $\tilde{\tau}^{ad}$ so that this term is always positive. Since the denominator is a squared term of a strictly negative quantity, the expression in Equation 10 is positive for all τ^{ad}. ∎

Proof of Lemma 5:

Proof is via the Implicit Function Theorem using the lobby's first order condition, Equation 5, referred to here as FOC_L.

$$\frac{\partial e}{\partial \tau^{ad}} = -\frac{\frac{\partial FOC_L}{\partial \tau^{ad}}}{\frac{\partial FOC_L}{\partial e}} = -\frac{\frac{\partial r}{\partial e}\frac{\partial \pi(\tau^{ad})}{\partial \tau^{ad}} + \frac{\partial^2 r}{\partial e \partial \tau^{ad}}\left[\pi(\tau^{ad}) - \pi(\tau^{a})\right]}{\frac{\partial^2 r}{\partial e^2}\left[\pi(\tau^{ad}) - \pi(\tau^{a})\right]}$$

The denominator is again negative.

Turning to the numerator: $\frac{\partial r}{\partial e}$ is positive and $\frac{\partial \pi(\tau^{ad})}{\partial \tau^{ad}}$ is positive by construction so the first term is positive.

We can rewrite $\frac{\partial r(e,\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}} = -\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))}{\partial \tau^{ad}} = -\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))}{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}\frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}} = -f_\gamma \frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}}$, where $F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))$ is the CDF of γ and $f_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))$ is the pdf of γ and $Z(\boldsymbol{\tau^a},\tau^{ad})$ again represents the left hand side of the inequality in Expression 8.

Then $\frac{\partial^2 r}{\partial e \partial \tau^{ad}} = -\frac{\partial}{\partial e}\left(\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))}{\partial \tau^{ad}}\right) = -f_\gamma \frac{\partial^2 Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial e \partial \tau^{ad}} - \frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}}\frac{\partial f_\gamma}{\partial e} = -\frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}}\frac{\partial f_\gamma}{\partial e}$ where the above equality holds because $Z(\boldsymbol{\tau^a},\tau^{ad})$ does not depend on e. $\frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}} \geq 0$ by the proof of Lemma 4, so $\frac{\partial f_\gamma}{\partial e} \geq 0$ ensures that $\frac{\partial^2 r}{\partial e \partial \tau^{ad}} \leq 0$. Thus the sign of $\frac{\partial e}{\partial \tau^{ad}}$ is ambiguous. ∎

Proof of Result 3:

$\frac{\partial R(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}} = \frac{\partial r}{\partial e}\frac{\partial e}{\partial \tau^{ad}} + \frac{\partial r}{\partial \tau^{ad}}$. $\frac{\partial r}{\partial e}$ is non-negative by Lemma 3. $\frac{\partial r}{\partial \tau^{ad}}$ is non-positive by Lemma 4. $\frac{\partial e}{\partial \tau^{ad}}$ is ambiguous by Lemma 5. Thus the sign of $\frac{\partial R(\boldsymbol{\tau^a},\tau^{ad})}{\partial \tau^{ad}}$ may be either positive, zero or negative. ∎

Proof of Lemma 6:

The left side of the inequality in Expression 8, $Z(\boldsymbol{\tau^a},\tau^{ad})$, does not depend on e. Thus we have $r(e,\boldsymbol{\tau^a},\tau^{ad}) = \Pr[Z < \gamma(e,\theta)] = 1 - F_\gamma(\gamma = Z)$ where $F_\gamma(\gamma = Z) = F_\theta(\theta = h^{-1}(\gamma, e))$ by the Change of Variables Theorem and Assumption 2 with $\gamma = h(e,\theta)$ giving the change of variable.

Then $\frac{\partial r}{\partial \gamma(\cdot)} = -\frac{\partial F_\theta(\theta = h^{-1}(\gamma,e))}{\partial \theta}\frac{\partial h^{-1}(\gamma,e)}{\partial \gamma(\cdot)} = -f_\theta(\theta)\frac{\partial h^{-1}(\gamma,e)}{\partial \gamma(\cdot)}$. Because $\gamma = h(e,\theta)$ is increasing in both e and θ by Assumption 1, $h^{-1}(\gamma,e)$ decreases when $\gamma(\cdot)$ increases for all (e,θ) pairs. The pdf of θ is non-negative, so $\frac{\partial r}{\partial \gamma(\cdot)} \geq 0$. ∎

Proof of Lemma 7:

Proof is via the Implicit Function Theorem for Banach spaces using the lobby's first order condition, Equation 5, referred to here as FOC_L.

$$\frac{\partial e}{\partial \gamma(\cdot)} = -\frac{\frac{\partial FOC_L}{\partial \gamma(\cdot)}}{\frac{\partial FOC_L}{\partial e}} = -\frac{\frac{\partial r}{\partial e}\left(\frac{\partial}{\partial \gamma(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]\right) + \frac{\partial^2 r}{\partial e \partial \gamma(\cdot)}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right]}{\frac{\partial^2 r}{\partial e^2}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right]}$$

The denominator is negative as show in the Proof of Lemma 2. The first term in the numerator is zero since the profit function does not vary in $\gamma(\cdot,\cdot)$. Combining this with the leading negative sign and the fact that $\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$ is positive, the sign of $\frac{\partial e}{\partial \gamma(\cdot)}$ is determined by the sign of $\frac{\partial^2 r}{\partial \gamma(\cdot)\partial e}$.

From the proof of Lemma 6, we have $\frac{\partial r(e,\boldsymbol{\tau^a},\tau^{ad})}{\partial \gamma(\cdot)} = -f_\theta(\theta)\frac{\partial h^{-1}(\gamma,e)}{\partial \gamma(\cdot)}$, where $F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))$ is the CDF of γ and $f_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))$ is the pdf of γ and $Z(\boldsymbol{\tau^a},\tau^{ad})$ again represents the left hand side of the inequality in Expression 8.

Then $\frac{\partial^2 r}{\partial e \partial \gamma(\cdot)} = -\frac{\partial}{\partial e}\left(f_\theta(\theta)\frac{\partial h^{-1}(\gamma,e)}{\partial \gamma(\cdot)}\right) = -f_\theta(\theta)\frac{\partial^2 h^{-1}(\gamma,e)}{\partial \gamma(\cdot)\partial e} - \frac{\partial h^{-1}(\gamma,e)}{\partial \gamma(\cdot)}\frac{\partial f_\theta}{\partial e} = -f_\theta(\theta)\frac{\partial^2 h^{-1}(\gamma,e)}{\partial \gamma(\cdot)\partial e}$ where the above equality holds because the pdf of θ does not depend on e. When $\gamma(\cdot)$ shifts up, $\frac{\partial h^{-1}(\gamma,e)}{\partial \gamma(\cdot)}$ becomes less negative, so $\frac{\partial^2 r}{\partial e \partial \gamma(\cdot)} \leq 0$. Thus $\frac{\partial e}{\partial \gamma(\cdot)} \leq 0$.

■

Proof of Result 4:

$\frac{\partial R(\boldsymbol{\tau^a},\tau^{ad})}{\partial \gamma(\cdot)} = \frac{\partial r}{\partial e}\frac{\partial e}{\partial \gamma(\cdot)} + \frac{\partial r}{\partial \gamma(\cdot)}$. $\frac{\partial r}{\partial e}$ is non-negative by Lemma 3. $\frac{\partial e}{\partial \gamma(\cdot)}$ is non-negative by Lemma 7. $\frac{\partial r}{\partial \gamma(\cdot)}$ is non-positive by Lemma 6. Thus the sign of $\frac{\partial R(\boldsymbol{\tau^a},\tau^{ad})}{\partial \gamma(\cdot)}$ can be positive, zero or negative. ■

Proof of Lemma 8:

The effect on the renewal probability is determined by the sign of the derivative of the left hand side of the inequality in Expression 8 with respect to a positive shift in the $\pi(\cdot)$ function; to show that the renewal probability is increasing in such a positive shift, I must demonstrate that the following derivative is negative:

$$\frac{\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)\left(\frac{\partial}{\partial \pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]\right) + ZN(\boldsymbol{\tau^a},\tau^{ad})\left(\frac{\partial}{\partial \pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]\right)}{\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)^2}.$$

This can be simplified to

$$\frac{\left[CS_X(\tau^{ad}) + TR(\tau^{ad}) - CS_X(\tau^a) - TR(\tau^a)\right]\left(\frac{\partial}{\partial \pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]\right)}{\left(\pi_X(\tau^a) - \pi_X(\tau^{ad})\right)^2}. \tag{11}$$

Assume the tariff that maximizes the term in square brackets is small enough so that this term is negative. This should hold almost everywhere, as the tariff that maximizes that term is zero in most cases, and certainly smaller than τ^O. Since the denominator is a squared term of a strictly positive quantity, the sign of the derivative is determined by the sign of $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$.

When $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right] \geq 0$ the derivative is weakly negative and $\frac{\partial r}{\partial\pi(\cdot)}$ is weakly positive.

When $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right] \leq 0$ the derivative is weakly positive and $\frac{\partial r}{\partial\pi(\cdot)}$ is weakly negative.

Thus the derivative of the renewal probability has the same sign as $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$, which indicates how the gap between profits under antidumping duties and profits under the trade agreement tariff changes when the profit function changes. ∎

Proof of Lemma 9:

Proof is via the Implicit Function Theorem for Banach spaces using the lobby's first order condition, Equation 5, referred to here as FOC_L.

$$\frac{\partial e}{\partial\pi(\cdot)} = -\frac{\frac{\partial FOC_L}{\partial\pi(\cdot)}}{\frac{\partial FOC_L}{\partial e}} = -\frac{\frac{\partial r}{\partial e}\left(\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]\right) + \frac{\partial^2 r}{\partial e\partial\pi(\cdot)}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right]}{\frac{\partial^2 r}{\partial e^2}\left[\pi(\tau^{ad}) - \pi(\tau^a)\right]}$$

Beginning with the denominator: $\tau^{ad} > \tau^a$ and $\pi(\cdot)$ is increasing in τ so the expression is brackets is positive. $\frac{\partial^2 r}{\partial e^2}$ is negative by Result 3, so the denominator is negative. Combining this with the leading negative sign, the sign of $\frac{\partial e}{\partial\pi(\cdot)}$ is determined by the sign of the numerator.

$\frac{\partial r}{\partial e}$ is positive by Lemma 3 and $\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$ is positive as argued above.

$\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$ may be either positive or negative, and its sign impacts $\frac{\partial^2 r}{\partial e\partial\pi(\cdot)}$. To see how, we can rewrite $\frac{\partial r(e,\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)} = -\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))}{\partial\pi(\cdot)} = -\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))}{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}\frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)} = -f_\gamma\frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)}$, where $F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))$ is the CDF of γ and $f_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))$ is the pdf of γ and $Z(\boldsymbol{\tau^a},\tau^{ad})$ again represents the left hand side of the inequality in Expression 8.

Then $\frac{\partial^2 r}{\partial e\partial\pi(\cdot)} = -\frac{\partial}{\partial e}\left(\frac{\partial F_\gamma(Z(\boldsymbol{\tau^a},\tau^{ad}))}{\partial\pi(\cdot)}\right) = -f_\gamma\frac{\partial^2 Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)\partial e} - \frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)}\frac{\partial f_\gamma}{\partial e} = -\frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)}\frac{\partial f_\gamma}{\partial e}$ where the above equality holds because $Z(\boldsymbol{\tau^a},\tau^{ad})$ does not depend on e. The proof of Lemma 8 shows that $\frac{\partial Z(\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)}$ has the opposite sign as $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$. When $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$ is weakly positive (negative), $\frac{\partial f_\gamma}{\partial e} \geq 0$ implies that $\frac{\partial^2 r}{\partial e\partial\pi(\cdot)} \geq 0$ (≤ 0). Thus $\frac{\partial e}{\partial\pi(\cdot)}$ has the same sign as $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$.

∎

Proof of Result 5:

$\frac{\partial R(\boldsymbol{\tau^a},\tau^{ad})}{\partial\pi(\cdot)} = \frac{\partial r}{\partial e}\frac{\partial e}{\partial\pi(\cdot)} + \frac{\partial r}{\partial\pi(\cdot)}$. $\frac{\partial r}{\partial e}$ is non-negative by Lemma 3. $\frac{\partial r}{\partial\pi(\cdot)}$ and $\frac{\partial e}{\partial\pi(\cdot)}$ are both non-negative (non-positive) when $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$ is non-negative (non-positive) by Lemmas 8 and 9. Thus the sign of the entire expression is the same as the sign of $\frac{\partial}{\partial\pi(\cdot)}\left[\pi_X(\tau^{ad}) - \pi_X(\tau^a)\right]$. ■

6 References

Bown, Chad P., "Taking Stock of Antidumping, Safeguards and Countervailing Duties, 1990-2009," The World Economy v34, n12 (December 2011): 1955-1998.